Διαγώνισμα Άλγεβρα Α' Λυκείου

20/12/2020

(Λύσεις)

Θέμα Α

(A1) Σχολικό βιβλίο, σελίδα 70

(A2) Σχολικό βιβλίο, σελίδα 71

(A3) i) Αδύνατη

ii) $\sqrt[\nu]{a^{\mu}}$

iii) $-\alpha$

(A4) 1. Σ

2. Λ

3. Σ

4. Λ

5. Σ

Θέμα Β

(B1) i) $A = \sqrt{8+\sqrt{5-\sqrt{16}}} =$

$= \sqrt{8+\sqrt{5-4}} =$

$= \sqrt{8+\sqrt{1}} =$

$= \sqrt{8+1} =$

$= \sqrt{9} =$

$= 3$

ii) $B = (3+\sqrt{27}-\sqrt{12})\cdot(\sqrt[3]{27}-\sqrt{48}+\sqrt{3}) =$

$= (3+\sqrt{9\cdot 3}-\sqrt{4\cdot 3})\cdot(3-\sqrt{16\cdot 3}+\sqrt{3}) =$

$= (3+3\sqrt{3}-2\sqrt{3})\cdot(3-4\sqrt{3}+\sqrt{3}) =$

$= (3+\sqrt{3})\cdot(3-3\sqrt{3}) =$

$= 9-9\sqrt{3}+3\sqrt{3}-3\sqrt{3}^2 =$

$= 9-6\sqrt{3}-9 =$

$= -6\sqrt{3}$

iii) $\Gamma = \sqrt{(1-\sqrt{3})^2} + \sqrt{(1+\sqrt{3})^2} =$

$= |1-\sqrt{3}| + |1+\sqrt{3}|$

Όμως $1<\sqrt{3}$ διότι $1^2 < \sqrt{3}^2 \Leftrightarrow 1<3$ ισχύει

Άρα $\Gamma = -1+\sqrt{3}+1+\sqrt{3} =$

$= 2\sqrt{3}$

B2 i) $\frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{3}+1} - \frac{3}{\sqrt{3}} = 0 \Leftrightarrow$

$\Leftrightarrow \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} + \frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)} - \frac{3\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = 0 \Leftrightarrow$

$\Leftrightarrow \frac{\sqrt{3}+\cancel{1}+\sqrt{3}-\cancel{1}}{\sqrt{3}^2-1^2} - \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = 0 \Leftrightarrow$

$\Leftrightarrow \frac{\cancel{2}\sqrt{3}}{\cancel{2}} - \sqrt{3} = 0 \Leftrightarrow$

$\Leftrightarrow \sqrt{3}-\sqrt{3} = 0$

$\Leftrightarrow 0=0$ ισχύει

ii) $1+\sqrt{2} > \sqrt{5} \Leftrightarrow$

$\Leftrightarrow (1+\sqrt{2})^2 > \sqrt{5}^2 \Leftrightarrow$

$\Leftrightarrow 1+2\sqrt{2}+\sqrt{2}^2 > 5 \Leftrightarrow$

$\Leftrightarrow 3+2\sqrt{2} > 5 \Leftrightarrow$

$\Leftrightarrow 2\sqrt{2} > 2 \Leftrightarrow$

$\Leftrightarrow \sqrt{2} > 1 \Leftrightarrow$

$\Leftrightarrow \sqrt{2}^2 > 1^2 \Leftrightarrow$

$\Leftrightarrow 2 > 1$ ισχύει

Θέμα Γ

Γ1) i) $|x-2| = 3-x$

Πρέπει $3-x \geq 0 \Leftrightarrow x \leq 3$

Οπότε $x-2 = 3-x$ ή $\cancel{x}-2 = \cancel{x}-3$

$2x = 5$ $\quad -2=-3$

$x = \frac{5}{2}$ $\quad$ Αδύνατη

ii) $\frac{4x-3}{x-1} - \frac{2x^2-1}{x^2-1} + \frac{2x+1}{-x-1} = 0$

Πρέπει $x-1 \neq 0$ κ' $x^2-1 \neq 0$ κ' $-x-1 \neq 0$

$x \neq 1$ $\quad (x-1)(x+1) \neq 0$ $\quad x \neq -1$

$x \neq 1$ κ' $x \neq -1$

$ΕΚΠ(x-1, x^2-1, x+1) = (x-1)(x+1)$

$\frac{4x-3}{x-1} - \frac{2x^2-1}{(x-1)(x+1)} - \frac{2x+1}{x+1} = 0 \Leftrightarrow$

$\Leftrightarrow \cancel{(x-1)}(x+1)\frac{4x-3}{\cancel{x-1}} - \cancel{(x-1)(x+1)}\frac{2x^2-1}{\cancel{(x-1)(x+1)}} - (x-1)\cancel{(x+1)}\frac{2x+1}{\cancel{x+1}} = 0 \Leftrightarrow$

$\Leftrightarrow \cancel{4x^2} - 3x + 4x - 3 - \cancel{2x^2} + 1 - \cancel{2x^2} - x + 2x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow 2x = 1 \Leftrightarrow$

$\Leftrightarrow x = \frac{1}{2}$

iii) $\frac{2-|x-2|}{3} - \frac{1-|4-2x|}{2} = |2-x| - \frac{8-|2-x|}{6}$

$ΕΚΠ(2,3,6) = 6$

$6 \cdot \frac{2-|x-2|}{3} - 6\frac{1-|2\cdot(2-x)|}{2} = 6|2-x| - 6\frac{8-|2-x|}{6} \Leftrightarrow$

$\Leftrightarrow 2\cdot(2-|x-2|) - 3(1-2|2-x|) = 6|2-x| - (8-|2-x|)$

$\Leftrightarrow 4-2|x-2|-3+\cancel{6|x-2|} = \cancel{6|x-2|}-8+|x-2| \Leftrightarrow$ (αφού $|a-b|=|b-a|$)

$\Leftrightarrow -2|x-2| - |x-2| = -4+3-8 \Leftrightarrow$

$\Leftrightarrow -3|x-2| = -9 \Leftrightarrow$

$\Leftrightarrow |x-2| = 3$

Άρα $x-2=3$ ή $x-2=-3$

$x=5$ ή $x=-1$

Γ2 $\lambda^2(x+4) - 5\lambda(x+\lambda) = -25$

$\lambda^2 x + 4\lambda^2 - 5\lambda x - 5\lambda^2 = -25$

$x \cdot (\lambda^2 - 5\lambda) = \lambda^2 - 25$

- Αν $\lambda^2 - 5\lambda \neq 0$ δηλαδή $\lambda(\lambda-5) \neq 0 \Leftrightarrow \lambda \neq 0$ κ' $\lambda \neq 5$

τότε η εξίσωση έχει μια λύση, την

$$x = \frac{\lambda^2-25}{\lambda^2-5\lambda} \Leftrightarrow x = \frac{(\lambda-5)(\lambda+5)}{\lambda(\lambda-5)} \Leftrightarrow x = \frac{\lambda+5}{\lambda}$$

- Αν $\lambda=0$ τότε η εξίσωση γίνεται:

$0 \cdot x = -25$

Αδύνατη

- Αν $\lambda = 5$ τότε η εξίσωση γίνεται:

$0 \cdot x = 0$

Αόριστη

Θέμα Δ

Δ1 $\lambda^2 \cdot (\lambda x - 1) + x - 2\lambda = 1 - 3\lambda x(\lambda+1) \Leftrightarrow$

$\Leftrightarrow \lambda^3 x - \lambda^2 + x - 2\lambda = 1 - 3\lambda^2 x - 3\lambda x \Leftrightarrow$

$\Leftrightarrow \lambda^3 x + 3\lambda^2 x + 3\lambda x + x = \lambda^2 + 2\lambda + 1 \Leftrightarrow$

$\Leftrightarrow x \cdot (\lambda^3 + 3\lambda^2 + 3\lambda + 1) = (\lambda+1)^2 \Leftrightarrow$

$\Leftrightarrow x \cdot (\lambda+1)^3 = (\lambda+1)^2$

Αφού η εξίσωση είναι ταυτότητα πρέπει:

$(\lambda+1)^3 = 0$ κ' $(\lambda+1)^2 = 0$

Άρα $\lambda = -1$

Οπότε $|(\omega+3)^{\lambda+2}| = 2021 \Leftrightarrow$

$\Leftrightarrow |(\omega+3)^{-1+2}| = 2021 \Leftrightarrow$

$\Leftrightarrow \quad |w+3| = 2021 \quad \Leftrightarrow$

$\Leftrightarrow \quad w+3 = 2021 \quad ή \quad w+3 = -2021$

$\Leftrightarrow \quad w = 2018 \quad ή \quad w = -2024$

(Δ2) i) $a = \sqrt[3]{4} \cdot \sqrt{\sqrt{2\sqrt[3]{2}}} =$

$= 4^{\frac{1}{3}} \cdot \sqrt{\sqrt{2 \cdot 2^{1/3}}} =$

$= (2^2)^{\frac{1}{3}} \cdot \sqrt{\sqrt{2^{1+1/3}}} =$

$= 2^{2/3} \cdot \sqrt{\sqrt{2^{4/3}}} =$

$= 2^{\frac{2}{3}} \cdot \sqrt[4]{2^{\frac{4}{3}}} =$

$= 2^{\frac{2}{3}} \cdot 2^{\frac{\frac{4}{3}}{\frac{4}{1}}} =$

$= 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} =$

$= 2^{\frac{2}{3}+\frac{1}{3}} =$

$= 2$

$$b = \frac{\sqrt[4]{\sqrt[3]{\sqrt[5]{4}}} - \sqrt[3]{\sqrt[5]{\sqrt{2}}} - 2}{\sqrt{2} \cdot \sqrt[3]{2\sqrt{2}}} =$$

$$= \frac{\sqrt[60]{4} - \sqrt[30]{2} - 2}{\sqrt{2} \cdot \sqrt[3]{2 \cdot 2^{\frac{1}{2}}}} =$$

$$= \frac{4^{\frac{1}{60}} - 2^{\frac{1}{30}} - 2}{\sqrt{2} \cdot \sqrt[3]{2^{3/2}}} =$$

$$= \frac{(2^2)^{\frac{1}{60}} - 2^{\frac{1}{30}} - 2}{2^{\frac{1}{2}} \cdot 2^{\frac{\frac{3}{2}}{\frac{3}{1}}}} =$$

$$= \frac{\cancel{2^{1/30}} - \cancel{2^{1/30}} - 2}{2^{1/2} \cdot 2^{1/2}} =$$

$$= \frac{-2}{2} =$$

$$= -1$$

ii) $\sqrt[3]{2a} \cdot \sqrt[3]{\sqrt{a-\beta} - \beta} \cdot \sqrt[3]{\sqrt{a-\beta} + \beta} =$

$= \sqrt[3]{2a} \cdot \sqrt[3]{(\sqrt{a-\beta} - \beta)(\sqrt{a-\beta} + \beta)} =$

$= \sqrt[3]{2a} \cdot \sqrt[3]{\sqrt{a-\beta}^2 - \beta^2} =$

$= \sqrt[3]{2a} \cdot \sqrt[3]{a - \beta - \beta^2} =$

$= \sqrt[3]{2 \cdot 2} \cdot \sqrt[3]{2 - (-1) - (-1)^2} =$

$= \sqrt[3]{4} \cdot \sqrt[3]{2} =$

$= \sqrt[3]{8} =$

$= 2$

Δ3) Αφού ισχύει $x^2 + y^2 = 1$ τότε

$x^2 = 1 - y^2$ κ' $y^2 = 1 - x^2$

Άρα $(x^2)^2 = (1-y^2)^2$ κ' $(y^2)^2 = (1-x^2)^2$

$x^4 = 1 - 2y^2 + y^4$ κ' $y^4 = 1 - 2x^2 + x^4$

Οπότε η παράσταση $\sqrt{x^4 + 4y^2} + \sqrt{y^4 + 4x^2}$ γίνεται:

$\sqrt{1 - 2y^2 + y^4 + 4y^2} + \sqrt{1 - 2x^2 + x^4 + 4x^2} =$

$= \sqrt{1 + 2y^2 + y^4} + \sqrt{1 + 2x^2 + x^4} =$

$= \sqrt{(1+y^2)^2} + \sqrt{(1+x^2)^2} =$

$= |1 + y^2| + |1 + x^2| =$

$= 1 + y^2 + x^2 + 1 =$ (αφού $1 + x^2 > 0$ κ' $1 + y^2 > 0$)

$= 1 + 1 + 1 =$

$= 3$